LITERATURE CITED

1. N. A. Fuks, Evaporation and Growth of Droplets in a Gaseous Medium [in Russian], Izd. Akad. Nauk SSSR, Moscow (1958).
2. Yu. M. Grigor'ev, B. I. Khaikin, N. M. Troyan, and A. G. Merzhanov, "Theory of droplet evaporation in a stationary medium," in: Heat and Mass Transfer [in Russian], Vol. 2, Minsk (1968), pp. 358-366.
3. J. Hubbart, V. Dennis, and A. Mills, "Droplet evaporation: effect of transients and variable properties," Int. J. Heat Mass Transfer, 18, 1003-1006 (1975).
4. S. V. Ananikov and A. V. Talanov, "Evaporation of a fuel droplet in a laminar gas flow," Fiz. Goreniya Vzryva, 9, No. 6, 849-855 (1973).
5. V. Ya. Rivkind and G. M. Ryskin, "Flow structure in the motion of a spherical droplet in a fluid medium in the interval of transition Reynolds numbers," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 1, 8-15 (1976).
6. V. Ya. Rivkind, G. M. Ryskin, and G. A. Fishbein, "Motion of a spherical droplet in a viscous fluid flow," Inzh.-Fiz. Zh., 20, No. 6, 1027-1035 (1971).
7. V. Ya. Rivkind, "Approximate methods for the solution of certain mass-transfer problems with chemical reaction," in: Theoretical Foundations of Chemical Technology [in Russian], GIPKh, Leningrad (1975), pp. 40-48.
8. B. I. Brounshtein and G. A. Fishbein, Hydrodynamics and Mass and Heat Transfer in Disperse Systems [in Russian], Khimiya, Leningrad (1977).

MATHEMATICAL MODELING OF A TRANSIENT HEAT-CONDUCTION PROCESS

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A numerical algorithm is proposed for solution of the transient heat-conduction equation by the Monte Carlo method. The calculated values of the temperature are compared with experimental data.

Let D be a finite domain of space (x, y, z) with boundaries Γ, $\bar{D} = D + \Gamma$. In the cylinder $\bar{Q}_T = \bar{D} \times [0 \le t \le T]$ with lateral surface $\Omega = \Gamma \times [0, T]$ it is required to find a solution U(x, y, z, t) of the problem

$$\frac{\partial U}{\partial t} = a\left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2}\right) + \varphi(x, y, z, t),\ (x, y, z) \in D,\ t > 0,$$
$$U(x, y, z, 0) = g(x, y, z),\ (x, y, z) \in \bar{D},$$
$$U|_\Omega = f(x, y, z, t),\quad (x, y, z) \in \Gamma,\quad 0 \leqslant t \leqslant T, \tag{1}$$

in which φ, f, and g are continuous functions and f(x, y, z, 0) = g(x, y, z) on Γ.

In the space (x, y, z) we introduce a uniform grid $(x_m, y_n, z_l) = (mh, nh, lh)$ with mesh spacing h, representing the set of points of intersection of the planes $x = mh$, $y = nh$, $z = lh$, where m, n, l are integers. Let ω_h be the set of interior nodes of the grid, i.e., nodes belonging to domain D, and let γ_h be the set of boundary nodes, i.e., nodes belonging to Γ or lying outside the domain $\bar{D}$ and situated at a distance from Γ smaller than the mesh spacing. Let $\bar{\omega}_h = \omega_h + \gamma_h$.

We introduce a grid with respect to the variable t: $\bar{\omega}_\tau = \{t_i = i\tau,\ i = \overline{0, k+1}\}$, where τ is the mesh spacing and $k = [T/\tau]$ is the integer part of the number T/τ. Let $\omega_\tau = \{t_i = i\tau,$ $i = \overline{1, k}\}$. We denote $\gamma_{h\tau} = \gamma_h \times \omega_\tau$, $\bar{\omega}_{h\tau} = \bar{\omega}_h \times \bar{\omega}_\tau$, $\omega_{h\tau} = \omega_h \times \omega_\tau$. The set of nodes of the grid $\bar{\omega}_{h\tau}$ situated in the hyperplane $t = t_i$ is called a layer.

Approximating the initial heat-conduction equation by an implicit computing grid and putting

$$\tau a / h^2 = 1, \tag{2}$$

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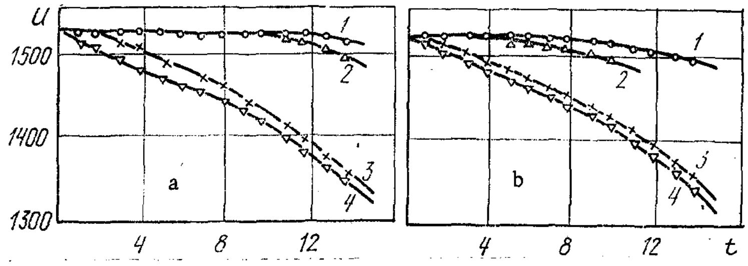

Fig. 1. Cooling temperature curves (U, °C, versus t, min) at the center (a) and at one fourth the length (b) of the casting. 1) Calculated by the proposed method; 2) experimental; 3) calculated according to [3]; 4) experimental on the surface.

we write problem (1) in the difference form

$$U^{i+1}_{m,n,l} = \frac{1}{7}\left(U^{i}_{m,n,l} + U^{i+1}_{m+1,n,l} + U^{i+1}_{m-1,n,l} + U^{i+1}_{m,n+1,l} + U^{i+1}_{m,n-1,l} + U^{i+1}_{m,n,l+1} + U^{i+1}_{m,n,l-1} + \tau\varphi^{i}_{m,n,i}\right), \tag{3}$$

$$(mh,\ nh,\ lh,\ (i+1)\tau) \in \omega_{h\tau},$$

$$U^{0}_{m,n,l} = g_{m,n,l},\quad (mh,\ nh,\ lh) \in \overline{\omega}_h,$$
$$U^{i}_{m,n,l} = f^{i}_{m,n,l},\quad (mh,\ nh,\ lh,\ i\tau) \in \gamma_{h\tau}. \tag{4}$$

The right-hand side of the second expression (4) is determined as follows: $f^{i}_{m,n,l}$ = f(mh, nh, lh, iτ) at a node belonging to Ω, and $f^{i}_{m,n,l}$ is taken to be equal to the value of f(x, y, z, t) at a point of Ω situated in the layer $t = t_i$ between the given node (mh, nh, lh, iτ) ∈ Ω and an interior node from which a wandering point escapes Ω. The right-hand side of the first expression (4) is determined analogously.

We consider the difference equation

$$U^{i+1}_{m,n,l} = \frac{1}{8}\left(U^{i+2}_{m,n,l} + U^{i}_{m,n,l} + U^{i+1}_{m+1,n,l} + U^{i+1}_{m-1,n,l} + U^{i+1}_{m,n+1,l}\right.$$
$$\left. + U^{i+1}_{m,n-1,l} + U^{i+1}_{m,n,l+1} + U^{i+1}_{m,n,l-1}\right),\ (mh,\ nh,\ lh,\ (i+1)\tau) \in \omega_{h\tau}. \tag{5}$$

If for a certain $i = \overline{0,\ k-1}$ we introduce the condition

$$\frac{U^{i+2}_{m,n,l} - U^{i+1}_{m,n,l}}{\tau} = \varphi^{i}_{m,n,l},\ (mh,\ nh,\ lh,\ (i+1)\tau) \in \omega_{h\tau}, \tag{6}$$

then in the domain $\omega_h \times t_{i+1}$ for the given $i = \overline{0,\ k-1}$ Eq. (5) goes over to (3), and (3) goes over to (5).

Thus, under condition (6) the computing schemes for problems (3), (4) and (5), (4), (6) coincide in the layer $t = t_{i+1}$, $i = \overline{0,\ k-1}$. Problem (3), (4) is therefore solved as a boundary-value problem for the Laplace difference equation by the Monte Carlo method with application of the direct modeling algorithm of [1, 2].

We compare the given algorithm for solution of the unsteady heat-conduction equation with the conventional procedure [3].

In [3] the solution of the difference boundary-value problem for the heat-conduction equation is approximately reduced to the solution of the Dirichlet problem for the Laplace difference equation. Since the computing schemes for the investigated problems do not coincide in this case, a loss of accuracy is suffered in the solution of the initial problem. In the proposed algorithm the computing schemes for the above-indicated difference boundary-value problems coincide. As a result, the accuracy of the solution can be greatly enhanced. For comparison of the algorithms (the comparison is made for the planar problem and $\varphi \equiv 0$, since the algorithm of [3] is applicable only in this case) the temperature fields are calculated, and the results of the calculations are compared with the experimental data. For specific problems with known solutions the error of calculation of the temperature fields by the proposed algorithm is smaller by 1/5 to 1/6 than by the conventional algorithm [3] for the same number of trials (number of paths of the wandering point), which is equal to 1000. Moreover, unlike the algorithm of [3], the investigated algorithm can be used to solve multidimensional problems.

Figure 1 gives an example of the calculation of the temperature field of a cylindrical casting of mild steel in a sand mold with a diameter of 120 mm (the experimental data are taken from [4]) by two procedures: the investigated algorithm and the one in [3]. The error of the solution obtained by the proposed procedure in comparison with the experimental data is 1.9%, and that of the procedure in [3] is 12%. The temperature values obtained by these two procedures are given in Fig. 1a for the center and in Fig. 1b for a quarter of the casting in the time interval from 0 to 15 min (initial temperature of the casting 1525°C).

Thus, the given algorithm can be used to plot with sufficient accuracy the unsteady temperature (transient) temperature fields of bodies of finite length with a complex cross-sectional configuration. An algorithm for the solution of multidimensional problems is synthesized by the indicated technique.

NOTATION

U, temperature; t, time; a, thermal diffusivity; x, y, z, coordinates; (x, y, z, t), temperature on the surface of the body; g(x, y, z), initial temperature; T, finite time of investigation of the process; $\varphi(x, y, z, t) = F(x, y, z, t)/c\rho$, where F(x, y, z, t) is an internal heat source; c, specific heat; ρ, density.

LITERATURE CITED

1. S. M. Ermakov and G. A. Mikhailov, Course in Statistical Modeling [in Russian], Nauka, Moscow (1976).
2. V. S. Gladkii, Probabilistic Computational Models [in Russian], Nauka, Moscow (1973).
3. N. P. Buslenko and Yu. A. Shreider, The Monte Carlo Method [in Russian], Fizmatgiz, Moscow (1961).
4. Hardening of Metals, Proc. Second Conf. Theory of Casting Processes [in Russian], Mashgiz, Moscow (1958).